The Lagrangian Relaxation Approach for Home Health Care Problems

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Abstract
Today, life expectancy and demand for caring for the elderly have been increased in developing countries. However, hospitals and nursing homes have not grown at the same rate. Thus, the home health care problem has been addressed by health academics and practitioners. The home care industry is a unique part of health care with growing demand, which has not been adequately addressed in the past. Based on the assumptions of this context and considering time windows and coordination constraints, a Lagrangian relaxation-based mixed-integer programming model is developed in this paper. To solve this model, since it is NP-hard, two metaheuristic algorithms, including simulated annealing and social engineering optimization, were applied. The results showed that the simulated annealing algorithm outperformed the other algorithm. Besides, a complete sensitivity analysis was performed to investigate the effect of the important parameters on the model outputs. As well as some perspectives on future works will be suggested.
Keywords: Home health care, Routing-scheduling problem, Time window, Lagrangian relaxation approach, metaheuristic algorithms

1 Introduction

Nowadays, the elderly and patients care is one of the main concerns for the World Health Organization (WHO) [31]. The current care demand is so high that the existing hospitals, health centers, and nursing homes cannot meet it. On the other hand, the elderly prefers to have self-care in their environment rather than be in a hospital. Home care is thus a solution to this problem.

The Homecare industry is a unique part of health care with growing demand, which has not been adequately addressed in the past. It faces technical challenges that other industries do not have [32, 33]. To meet their growing demand, home care organizations should optimize their activities. In this regard, planning and determining a nurse's route to visit patients at home is of particular importance, and ignoring it will lead to increased costs, which is of great importance to stakeholders, such as researchers, physicians, and policymakers [34, 35].

In this research, a new multi-purpose location-allocation-routing-scheduling model with a time window is developed, aiming at minimizing the travel cost and travel time of employees. Transportation cost constitutes a major cost of health care centers. To solve this model, the Lagrangian relaxation algorithm was applied to find upper and lower limits that simultaneously satisfy feasibility and optimality conditions. Due to the complexity and NP-hardness of the model, simulated annealing (SA) and social engineering optimizer (SEO) algorithms are used to solve the model. Finally, this model provides the best route and schedule for each employee through which the travel cost is minimized.

Generally, many of the basic models' in-home health care (HHC) problems are vehicle-routing problems. Cheng et al. [1] Investigated a vehicle routing problem with time window (VRPTW) including multiple depots. Considering both full-time and part-time nurses, they have optimized the cost of the working hours. As full-time nurses work overtime and part-time nurses are paid hourly, their objective was to minimize the amount of overtime and part-time work. Fenton et al. [2] studied the allocation and routing of HHC nurses in Germany. They used a two-stage approach to schedule the weekly service. They generated an initial solution using innovative approaches and then the scheduling was improved. Cappanera et al. [3] investigated the impact of politics on production planning and routing decisions in HHC. They used specific scenarios to integrate different decisions. Husebø et al. [4] provided a mixed-integer mathematical model for the routing and scheduling of HHC taking different treatments and time windows into account. Lanzarone et al. [5] proposed a solution to the assignment problem under the care continuity that minimizes nurses' maximum overtime. Yalçındağ et al. [6] considered routing and allocation problems in HHC and used the kernel regression method to estimate the travel time of a vehicle. Braekers et al. [7] addressed a multi-objective routing and scheduling problem in HHC, and to solve it, using a combination of multi-dimensional local search and large neighborhood search. Considering time windows and continuity of care, Valledor et al. [8] used an integer model based on the collection partitioning and neighborhood search. Decerle et al. [9] Used an integer model to incorporate soft time windows and staff competency. [10] Proposed a multi-objective model that minimizes care time and increases the quality of care. They used a memetic algorithm to solve their model. Lin et al. [11] proposed a two-objective model that the first model considered the positions of nurses and vehicles for cost reductions, and the second model expanded the first model considering the occurrence of sudden accidents and used the harmonic search algorithm.

Focusing on environmental pollution problems, Fathollahi-Fard et al. [12] presented a two-objective model and used a modified algorithm to solve the model. They conducted a sensitivity analysis to validate their model. Frazzon et al. [13] provided a flexible model using care prioritization to make the schedule more flexible and increase the number of visited locations. Fard et al. [14] studied an integrated model for basic decisions on allocation problem and used a competitive imperialist algorithm, well suited for large models, to solve the problem. Focusing on economic sustainability and operational efficiency and considering demand and capacity aspects, Nasir et al. [15] considered a MILP model for home care problems. They used a neighborhood search and compared the results with Cplex. Gong et al. [16] used a two-objective model to minimize organizational operations and maximize customer satisfaction. Mohammadi et al. [17] Modeled the home care problem as a covering problem and introduced uncertainty to the nursing network in Shiraz, Iran. Decerle et al. [18] investigated a home care problem and combined ant colony and memetic algorithms to solve it.

Grenouilleau et al. [19] looked at home care problems as a perspective robust optimization (RO). Specifically, based on the theory of budget uncertainty, they defined a model with uncertain variables, which should be resolved whenever a new person requested home care. They used taboo search and neighborhood search methods to solve the model and performed a series of experiments to validate the proposed models and algorithms. Erdem et al. [20] investigated a type of home care routing in which a group of healthcare workers performs the required number of jobs using electric vehicles. They explored a combination of genetic and neighborhood variable search algorithms to solve their proposed model [21, 22]. Regarding the literature of home care, a mixed-integer linear programming model based on Lagrangian relaxation is developed in this paper. As the model is NP-hard, simulated annealing (SA), and social engineering optimization (SEO) metaheuristic algorithms are used [23, 24].

This paper is organized as follows. Defining the parameters, variables, and constraints, the problem is modeled in Section 2. In Section 3 represents the numerical results of the experiments, a description of the test instances, and an analysis of the results. Finally, in Section 4 some concluding remarks, as well as some perspectives on the future works, are provided.

2 Problem definition

In this HHC problem, a collection of nurses and a set of required services are programmed such that the transportation cost and dissatisfaction of the patients and nurses are minimized. The working time of each nurse is a fixed value over the week and the home organization should pay them a certain wage [25, 26]. Each nurse is capable of doing some services, which should be considered in the allocation process. In each center, the nurses' time windows are known. Applying the nurses outside of their time windows gives rise to dissatisfaction [27]. They should be scheduled such that their dissatisfaction over the working load is minimized. The patients have also their time windows, where they will be satisfied if they are properly serviced in these time windows. These constraints should be met by the solutions of the model [28].

2.1 Soft time window and synchronization constraints

In this section, we explain the penalties of not being satisfied with soft time windows and coordination limitations [29]. The penalties are added to the objective function when the nurses are not satisfied with their time windows or the patients are too far apart or when the patients receive care outside their time windows [9, 30]. The penalty for the patient *i* is denoted by $f_i(t_i)$. The amount of penalty imposed depends on the time the care of the patient *i* starts. The penalty growth coefficient in the normal range is v_i^t , while it is multiplied by p^t if the care is outside of the normal range. The value of the normal and out of normal time for patient *t* is given by τ_i and c_i represents the amount of time needed to perform the care (visit time). We consider that $\tau_i = \lambda \times c_i$, where λ is the conformance percentage between the visit time and the time window of a patient. Accordingly, the penalty function is defined as follows:

$$f_i(t_i)=\begin{cases}(\tau_i+(a_i-t_i-\tau_i)\times p^t)\times v_i^t, if\ t_i<a_i-\tau_i\\(a_i-t_i)\times v_i^t, if\ t_i\in[a_i-\tau_i, a_i]\\0, if\ t_i\in[a_i, b_i]\\(t_i-b_i)\times v_i^t, if\ t_i\in[b_i, b_i+\tau_i]\\\left(\tau_i+(t_i-b_i-\tau_i)\times p^t\right)\times v_i^t, if\ t_i>b_i+\tau_i\end{cases} \tag{1}$$

Also, the penalties imposed on the non-concordance between the two nurses who should take care of a patient is shown by $g_{i,j}(t_i,t_j)$. To separate the normal and non-normal regions of two nurses *i* and *j* who have a joint visit, τ_{ij} is used. The rate of the concordance was previously denoted by λ, and accordingly, we have $\tau_{ij} = \lambda \times c_i$. The penalty for the normal region is shown by is v_{ij}^s. If a nurse arrival time is greater than τ_{ij}, the penalty is multiplied by p^s [9, 36].

$$g_{i,j}(t_i,t_j)=\begin{cases}(\tau_{i,j}+p^s\times(|t_i-t_j|-\tau_{i,j}))\times v_{i,j}^s, if\ |t_i-t_j|>\tau_{i,j}\\|t_i-t_j|\times v_{i,j}^s, if\ |t_i-t_j|\le t_{i,j}\\0, if\ t_i=t_j\end{cases} \tag{2}$$

2.2 Mathematical model

In this section, the mathematical model is formulated based on a routing and scheduling problem in the home health care network. Then, the notations of the proposed model are reported as follows:

N	Set of nodes
O	Set of visits
S	Set of staff members
P	Set of HHC offices
R	Set of job roles
$Sync$	Set of synchronized visits
$[i^t, u^t]$	Planning period of time
d_{ij}	Distance between the nodes *i* and *j*
c_i	Duration of the visit *i*
$[\alpha_i, \beta_i]$	Working time window of the staff member *i*
$[a_i, b_i]$	Availability time window of the patient *i*
η_i	Qualification of the staff member *i*
ρ_i	Qualification level required to perform the visit *i*
y_i^k	Association of the staff member *i* to the office *k*
δ_{ij}	Indicates whether *i* and *j* are synchronized visits or not

p^{t}	The coefficient for time windows dissatisfaction
p^{s}	The coefficient for non-synchronized visits dissatisfaction
π^{t}_{ijn}	Lagrangian Relaxation Coefficient
λ	The percentage used for the penalties computation
M	A large number
x^{k}_{ij}	If the employee k travels from i to *j*, it equals 1, otherwise it is 0.
t_i	The start time of visiting i.

The HHC routing and scheduling problem is formulated as a Lagrangian relaxation mixed-integer programming model as follows:

$$\min \sum_{i,j\in N}\sum_{K\in s} x_{ij}^{k}\times d_{ij} + \sum_{i\in o} f_i(t_i) + \sum_{i,j\in o}\delta_{ij}\times g_{ij}(t_i,t_j) + \pi^{t}_{ijn}\left(\begin{array}{l}\left(\sum\sum t_i - t_j + c_i + d_{ij} + \left(\left(\sum x_{ij}^{k}\right)-1\right)\times m\right) + \left(\sum\sum \alpha_k - t_i\left(\left(\sum x_{ij}^{k}\right)-1\right)\right) \\ + \left(\sum\sum\sum t_i - \beta_k + c_i + d_{ij} + (x-1)\right)\times m\end{array}\right) \tag{3}$$

s.t:

$$\sum_{\substack{j\in n\\ i\neq j}}\sum_{k\in s} x_{ij}^{k} = 1 \tag{4}$$

$$\sum_{j\in o} x_{ij}^{k} = \sum_{j\in o} x_{ji}^{k} = \gamma_{k}^{i}, \forall i\in p, k\in s \tag{5}$$

$$\sum_{\substack{j\in n\\ i\neq j}} x_{ij}^{k} = \sum_{\substack{j\in n\\ i\neq j}} x_{ji}^{k}, \forall i\in n, k\in s \tag{6}$$

$$\sum_{\substack{j\in n\\ i\neq j}} x_{ij}^{k} = 0, \forall i\in o, k\in s. \eta_k \neq \rho_i \tag{7}$$

$$x_{ij}^{k}\in\{0,1\}, \forall i\in N, j\in N, k\in S, t_i\in T, i\in o \tag{8}$$

Regarding the time windows penalties and coordinator constraints, the objective function (3) minimizes the total time of traveling to patient's homes, visiting them in their time windows, and returning to home care center penalties. Constraint (4) ensures that each visit is made by only one person. Constraint (5) examines the staff entry and exit. Constraint (6) regulates the flow of nurses. In Constraint (7), the employees' qualification for visiting a certain patient is evaluated. Constraint (8) specifies the variable domain. Constraints (1) and (2) are used to linearize the model.

3 Numerical experiments

This section examines the proposed algorithms and compares them with each other. The proposed algorithms were implemented in MATLAB R2016a using a computer with an Intel i7 1.8 GHz processor and 4GB of RAM under Windows 10 environment.

Table 1. The result of metaheuristic algorithms (SA-SEO)

	Best cost			Best cost	
Iteration	SA	SEO	Iteration	SA	SEO
1	1030.88	66436.26	26	680.66	7577.97
2	961.70	53899.97	27	680.66	7022.26
3	850.43	42058.14	28	680.66	6899.98
4	753.34	37984.77	29	680.66	6605.31
5	680.66	37592.14	30	680.66	6412.61
6	680.66	34816.33	31	680.66	6412.61
7	680.66	33669.19	32	680.66	6412.61
8	680.66	28756.87	33	680.66	5341.07
9	680.66	28477.25	34	680.66	4846.51
10	680.66	23611.77	35	680.66	4846.51
11	680.66	22890.19	36	680.66	4647.04
12	680.66	20340.99	37	680.66	4635.08
13	680.66	16342.68	38	680.66	4625.01
14	680.66	15008.82	39	680.66	4271.89
15	680.66	14876.09	40	680.66	4028.95
16	680.66	13933.85	41	680.66	3900.29
17	680.66	13353.31	42	680.66	3631.47
18	680.66	12853.43	43	680.66	3462.97
19	680.66	12262.92	44	680.66	3288.80
20	680.66	11641.61	45	680.66	3199.24
21	680.66	11478.63	46	680.66	3199.24
22	680.66	11353.01	47	680.66	3183.90
23	680.66	10071.81	48	680.66	2996.36
24	680.66	7884.06	49	680.66	2802.44
25	680.66	7725.33	50	680.66	2761.59

For both algorithms, 50 iterations were considered. The obtained results are shown in Table 1. As can be seen, for the simulated annealing algorithm, the best solution started at 1030.88, and after 50 replicates reached 680.66. For the social-engineering algorithm, the best solution in the first replicate was 66436.26 and after 50 replicates reached 2761.59. Their corresponding diagrams are shown in Fig.9.The performance of both algorithms for eight samples with different parameters are presented in Table.2.

Table 2. Parameter tuning for the SEO and SA algorithms

Algorithms	Parameters	Parameter tuning combinations							
		1	2	3	4	5	6	7	8
SA	Max-iteration	100	200	300	400	600	800	900	1000
	Sub-iteration	50	150	250	350	450	550	650	750
	Mutation	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55
	Initial temperature	0.3	0.35	0.45	0.55	0.65	0.75	0.85	0.95
SEO	Max-iteration	200	300	400	600	800	900	1000	200
	Collecting data	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55

	Connecting attacker	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
	Connections	50	75	100	125	150	175	200	225

According to this table, the SEO algorithm outperforms the SA algorithm. The best solution of the SA algorithm was 680, while the SEO algorithm was able to get to 1095 in replicate 249. The mean and standard deviation of the results are shown in Table 3.

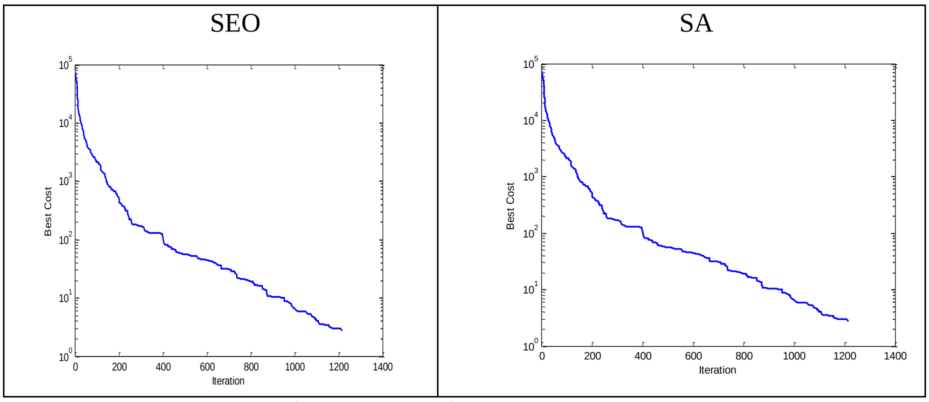

Fig.9. The objective function values for the proposed SA and SEO algorithms

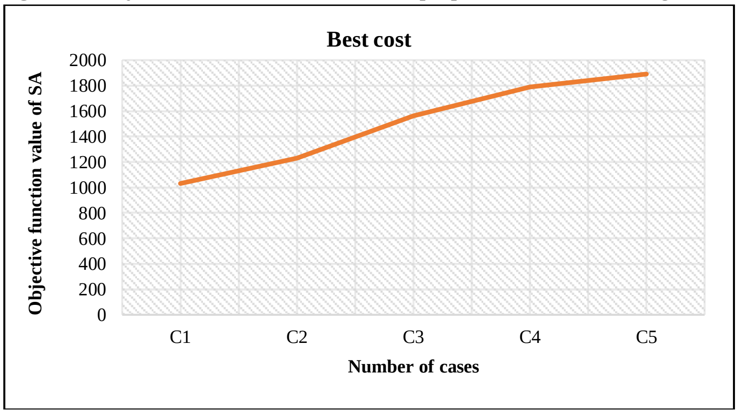

Fig. 10. The objective function values for different number of cases

Table 3. The average and STDEV of the proposed algorithms results in 100 iterations

Algorithm	AVERAGE	STDEV
SA	689.3977	48.002
SEO	7928.525	11861.61

As evident in Table 3, the solutions obtained by the SA algorithm have a lower mean and standard deviation. The problem has been considered in small, medium, and large scales, the setup of which are given in Table 4.

Table 4. The setup of problem sizes and levels.

Problem levels	Problem size (N/O/S/P/R)
Small scale	1(2/3/2/2) 2(2/3/4/1) 3(1/2/3/4)
Medium scale	4(3/3/5/6) 5(3/5/6/7) 6(5/5/7/6)
Large scale	7(6/5/6/7) 8(7/7/8/5) 9(9/7/10/9)

3.1 Sensitivity analyses for the HHC model developed

To investigate the behavior of the HHC model, a sensitivity analysis was performed on the important parameters of the model. In this regard, a series of changes were introduced to the model parameters, including the number of nurses (*s*), number of patients (*n*), and number of HHC centers (*p*). Each analysis is performed for five samples C1 to C5. The results are reported in Tables 5 to 7 and Fig.9 to 11.

Table 5 shows the results of the sensitivity analysis of the number of employed nurses (*s*). The objective function behavior is shown also in Fig. 10. Table 6 shows the results of the sensitivity analysis on the number of patients served (n). The objective function behavior is clearly displayed in Fig. 11. Finally, Table 7 shows the results of the sensitivity analysis on the number of HHC centers (p), and Fig. 12 illustrates the objective function behavior. The sensitivity analysis of the number of nurses reveals that as the number of nurses increases, the total cost increases. As the number of patients increases, the objective function naturally increases because the number of times patients are visited increases.

Table 5. The sensitivity analyses on the number of employed nurses

Number of cases	S	Best cost
C1	3	1030.88
C2	4	1231.54
C3	5	1560.12
C4	6	1788.22
C5	7	1890.34

Table 6. The sensitivity analyses on the number of patients served

Number of cases	N	Best cost
C1	9	1030.88
C2	11	1567.38
C3	13	1987.25
C4	15	2131.53
C5	17	2431.43

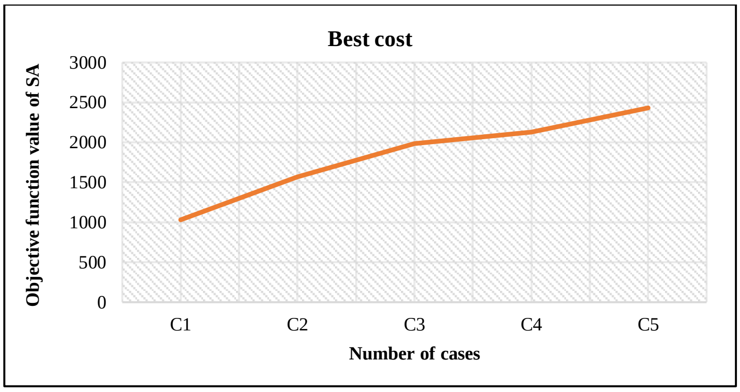

Fig. 11. The objective function values for different number of cases

As the number of home care centers increases, the objective function also increases. By the opening of home care centers, the access and visit time are reduced.

Table 7. The sensitivity analyses on the number of HHC centers

Number of cases	P	Best cost
C1	4	1030.88
C2	6	1621.07
C3	8	1745.21
C4	10	1978.51
C5	12	2014.32

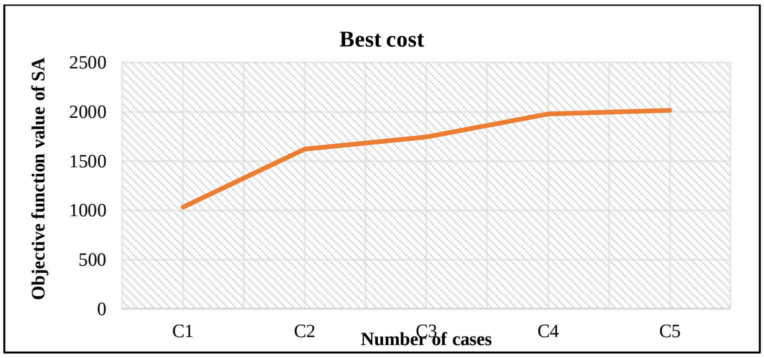

Fig. 12. The objective function values for different number of cases

4 Conclusions

In this paper, a new location-allocation-routing-scheduling model with a time window and coordination constraints was developed to minimize the travel cost, which is one of the most important costs of the system, and travel time of employees. Due to the complexity and NP-hardness of the model, the metaheuristic SA and SEO algorithms were used to solve the model. The results illustrated that the SA algorithm outperformed the SEO algorithm in terms of standard deviation. The results of the sensitivity analysis revealed that an increase of the patients imposed more cost to the system and reduces the accessibility to nurses, while when the number of HHC centers increased, easier accessibility or better coverage was obtained at the price of more cost.

In future research, the model can be developed dynamically, where new patients can enter the model or some requests can be canceled. Also, to make the model closer to the real world, the travel time of nurses can be assumed probabilistic depending on traffic volume. This can increase the efficiency in the system.

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